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Comparison of Linear Stability Results with Flight Transition Data

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Introduction

A NEED exists to predict the location of transition on aerodynamic surfaces both efficiently and with reasonable accuracy. Currently, the most common approach for predicting transition in two- and three-dimensional flows is the empirical e^N method,[1,2] which utilizes linear stability theory. The e^N method assumes that transition takes place when the integrated growth rate computed from linear stability theory reaches a certain value N. As a result, this method is most successful in predicting transition when most of the disturbance growth that leads to transition is linear. Quiet wind tunnels and actual flight conditions are typical environments that allow such linear growth of disturbances over long distances before transition onset. Under these conditions, transition onset can be correlated with an N factor on the order of 9–11.[3] On the other hand, noisy tunnels introduce disturbances with large initial amplitudes, and the associated transition process is such that its prediction is outside the scope of the e^N method.

In this work, we compare the results of the e^N method with the experimental flight data of Fisher and Dougherty[4] for compressible flow past a sharp cone. The comparisons demonstrate the effect of mild heat transfer at subsonic freestream Mach numbers and the effect of compressibility for freestream Mach numbers up to 2.

Masad and Malik[5] developed a transition correlation based on the e^9 method for subsonic flow over a flat plate. The correlation accounts for the effects of uniform wall suction, heat transfer, and Mach number. The correlation is given by

$$(Re_x)_{N=9} = a^2\Lambda^2 \tag{1a}$$

$$\Lambda = \Lambda_1\Lambda_2\Lambda_3 \tag{1b}$$

where

$$\Lambda_1 = a_1 + M_\infty^{a_2} + a_3 M_\infty^{a_4} \tag{2}$$

$$\Lambda_2 = b_1 + 10^8|v_w|^{b_2} + b_3 10^8|v_w|^{b_4} \tag{3}$$

$$\Lambda_3 = c_1 + (T_{\text{ad}}/T_w)^{c_2} + c_3(T_{\text{ad}}/T_w)^{c_4} \tag{4}$$

and

$$a = 0.0183, \quad a_1 = 4.1, \quad a_2 = 3.5$$

$$a_3 = 1.41, \quad a_4 = 1.83$$

$$b_1 = 1310, \quad b_2 = 1.28, \quad b_3 = 0.082, \quad b_4 = 1.0$$

$$c_1 = 7.9, \quad c_2 = 5.8, \quad c_3 = 9.1, \quad c_4 = 5.9$$

In Eqs. (1–4), Re_x is the x Reynolds number given by $Re_x = U_\infty^* x^*/\nu_\infty^*$, where U_∞^* is the freestream velocity, x^* is the distance measured from the leading edge of the flat plate, and ν_∞^* is the freestream kinematic viscosity. The continuous uniform nondimensional suction velocity is denoted by v_w such that $v_w = v_w^*/U_\infty^*$, where v_w^* is the dimensional suction velocity. The actual nondimensional wall temperature is denoted by T_w, whereas the adiabatic nondimensional wall temperature is denoted by T_{ad}. Both T_w and T_{ad} are made nondimensional with respect to the dimensional freestream temperature T_∞^*, such that $T_w = T_w^*/T_\infty^*$ and $T_{\text{ad}} = T_{\text{ad}}^*/T_\infty^*$. The ratio T_w/T_{ad} is used to specify the level of heat transfer. For a value of T_w/T_{ad} that is less than unity, the plate is cooled; when the value is larger than unity the plate is heated. In Eq. (2), M_∞ is the freestream Mach number. In the calculations used to develop the previous correlation and throughout this work, the specific heat at constant pressure is assumed to be a constant. The Prandtl number Pr is also assumed to be fixed at 0.72. The variation of dynamic viscosity with temperature is given by the Sutherland formula, and the freestream temperature is 300 K. The effect of variation in the freestream temperature on transition location is insignificant for subsonic flow. For example, at Mach 0.8, the predicted transition Reynolds number changed from 5.57×10^6 to 5.48×10^6 when T_∞^* decreased from 300 K to 150 K.

Equations (1–4) determine that the ratio r of $(Re_x)_{N=9}$ with heat transfer to $(Re_x)_{N=9,\text{ad}}$ for the adiabatic plate is given by

$$r = \frac{(Re_x)_{N=9}}{(Re_x)_{N=9,\text{ad}}} = \frac{\Lambda_3^2}{\Lambda_3^2|_{\text{ad}}}$$

or

$$r = \frac{\Lambda_3^2}{324} \tag{5}$$

A comparison of the results of Eq. (5) with the experimental flight data of Fisher and Dougherty[4] is shown in Fig. 1. In the figures the predicted transition Reynolds number $(Re_x)_{N=9}$ is denoted by Re_t. Fisher and Dougherty's curvefit of the experimental data is shown for the variation of r with T_w/T_{ad}; the agreement between our correlation and the curvefit is remarkably good over most of the range. The experimental data in Fig. 1 are for freestream Mach numbers in the range from 0.55 to 0.86. Fisher and Dougherty[4] noted no significant dependence of r on M_∞. Our correlation for r [Eq. (5)] has no dependence on M_∞ either. The correlation results are for flow over a flat plate, and the experimental data are for flow past a cone. It is assumed that the cone transition Reynolds number

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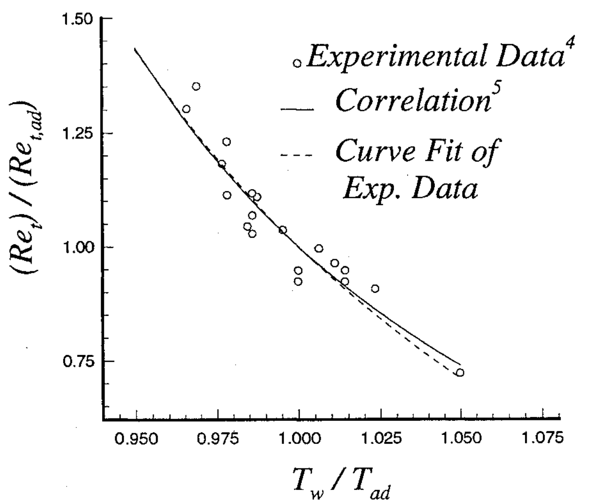

Fig. 1 Variation of ratio of transition Reynolds number with heat transfer to transition Reynolds number at adiabatic conditions with level of heat transfer.

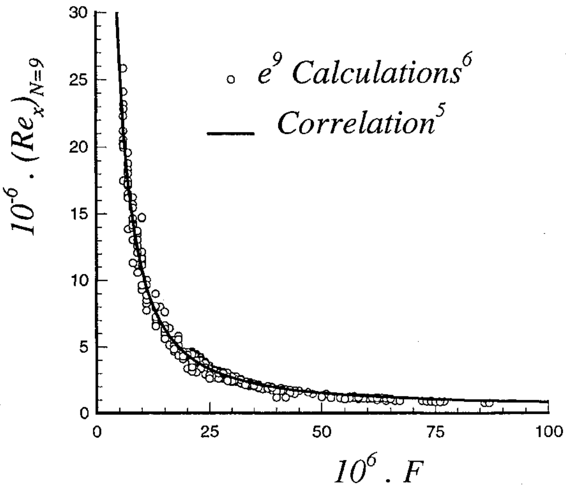

Fig. 2 Variation of predicted transition Reynolds number with frequency predicted to be causing transition using e^9 method.

is related to the flat-plate transition Reynolds number through a constant, and therefore the ratio r is the same for both flat plate and a cone.

The disturbance frequency F associated with e^9 amplification was correlated by Masad and Malik[5] with the predicted transition Reynolds number as

$$(Re_x)_{N=9} = 254\Gamma \tag{6}$$

where

$$\Gamma = 1700 + \frac{10^6}{(F \times 10^6)^{1.39}}$$

In Eq. (6), the frequency F is given by $2\pi f^* \nu_\infty^*/U_\infty^{*2}$, where f^* is the dimensional frequency of the disturbance in cycles per second (Hz). Correlations (1) and (6) are for flow over a smooth flat plate with $0 \le M_\infty \le 0.8, 0.95 \le (T_w/T_{ad}) \le 1.05, 0 \le -v_w \le 2 \times 10^{-5}$ (suction only), and $10 < F \times 10^6 < 31$.

The correlation of F with $(Re_x)_{N=9}$ given by Eq. (6) was developed for the conditions and the ranges stated earlier. However, the results indicate that this correlation is applicable under more general conditions and over a wider range of parameters. We compared the results of the correlation [Eq. (6)] with the theoretical results for flow over both smooth surfaces and rough surfaces that might induce separation (i.e., a surface with a hump[6]). We varied the unit Reynolds number; the hump height, length, shape, and location; the freestream Mach number M_∞ up to 0.8; the continuous uniform suction level up to $v_w = -9 \times 10^{-4}$; the levels of continuous cooling up to $(T_w/T_{ad}) = 0.8$; and the location of the suction and the heat transfer strip. Over 650 combinations of these conditions and parameters were used; for each combination, the predicted transition location was calculated, based on the e^9 method. The comparison between the correlation and the direct computations is shown in Fig. 2. The agreement is remarkably good. Note that for the data used in the development of the correlation (6), the frequencies ranged from $F \times 10^6 = 10\text{–}31$, whereas in the direct computations in Fig. 2 they range from $F \times 10^6 = 6\text{–}87$.

We also compared our results from correlation (1) and from the e^9 method with the experimental flight data of Fisher and Dougherty,[4] for freestream Mach numbers in the range from 0.5 to 2. Here, correlation (1) was used up to a freestream Mach number of 0.8, and then results from e^9 calculations were used in which the obliqueness of the most amplified first mode[7] was taken into account, under the conditions of adiabatic wall and no suction. The comparison is made between the experimental and theoretical ratios of the transition Reynolds number at any Mach number to the transition Reynolds number at $M_\infty = 1$. This is done to compare our flat-plate results with the experimental data for a cone. By com-

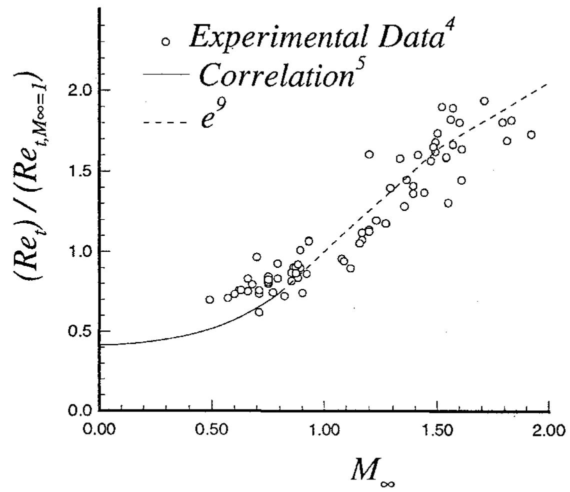

Fig. 3 Variation with freestream Mach number of ratio of transition Reynolds number at any Mach number to transition Reynolds number at a Mach number of unity.

paring such ratios, we are assuming that over the considered range of Mach numbers the cone transition Reynolds number is related to the flat-plate transition Reynolds number through a constant. Such an assumption was also made in the comparison in Fig. 1. Chen et al.[8] have shown that the ratio between the cone transition Reynolds number and the flat-plate transition Reynolds number in wind-tunnel tests can vary significantly, depending on the tunnel noise. The comparison is shown in Fig. 3; the agreement is very good. For the range of Mach numbers shown in the figure, it was found that the flat-plate e^9 calculations correlate the cone flight data if it is assumed that $(Re_t)_{\text{plate}}/(Re_t)_{\text{cone}} = \sqrt{3}$. Chen et al. found the ratio to be about $\sqrt{2.4}$ for Mach 3.5 under low-disturbance conditions.

Acknowledgments

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Spurious Numerical Solutions in Higher Dimensional Discrete Systems

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Introduction

IN the present paper, we discuss the qualitative features of spurious asymptotes which are found as numerical solutions on discretizing the original continuous partial differential equation (PDE). In the recent literature,[1,2] it is reported that chaotic solutions, even when the true solutions of the original differential equation approach limit cycles or fixed points, are often obtained as a consequence of the omission of the local discretization errors in the approximation of continuous differential equations by their discretized counterparts. The original and systematic work of Refs. 4–6 suggests that investigating typical features of such stable and unstable spurious numerical solutions (periodic points, limit cycles, tori, and chaotic motions), alias ghost solutions, not only could contribute to interpretations of numerical results in fluid dynamic study but may also provide useful knowledge about convergence speed for such steady-state solutions. This study presents analysis concerning behavior of spurious numerical solutions on applying a nonlinear dynamics approach. This supports the original work of Refs. 3–6. Furthermore, we discuss the dependence of difference schemes on nonlinear instability in cases of higher dimensional dynamical systems. The one-dimensional Burgers equation was selected for the analysis of partial differential equations representative of fluid dynamics.

Analysis

We consider the initial boundary value problem for the one-dimensional Burgers equation and its finite difference approximation. In this problem, in which boundary values are zero (fixed), only one trivial solution $[u(x) = 0]$ is allowed with the physical meaning.[7] Central spatial difference and Euler forward temporal difference schemes are applied

$$\begin{cases} u_i^{m+1} = u_i^m + \Delta t\left\{ -u_i^m \dfrac{u_{i+1}^m - u_{i-1}^m}{2\Delta x} + \nu \dfrac{u_{i+1}^m - 2u_i^m + u_{i-1}^m}{(\Delta x)^2} \right\} \\ \qquad [0 < i < (n-1), m \geq 0] \\ u_i^0 : \text{ initial data} \\ u_0^m = u_{n-1}^m = 0 \end{cases} \tag{1}$$

Figure 1 shows a comparison of bifurcation diagrams that result on using the spatial central difference scheme (1), where $\nu = \frac{1}{4}$ and $\Delta x = \frac{1}{8}$ are fixed. Initial data are $u_0^0 = 0.0$, $u_1^0 = 0.5$, $u_2^0 = 2.0$, $u_{n-2}^0 = -1.45$, $u_{n-1}^0 = 0.0$, and $u_i^0 = 2.0 - 3.45 \times (i-2)/(n-4)$

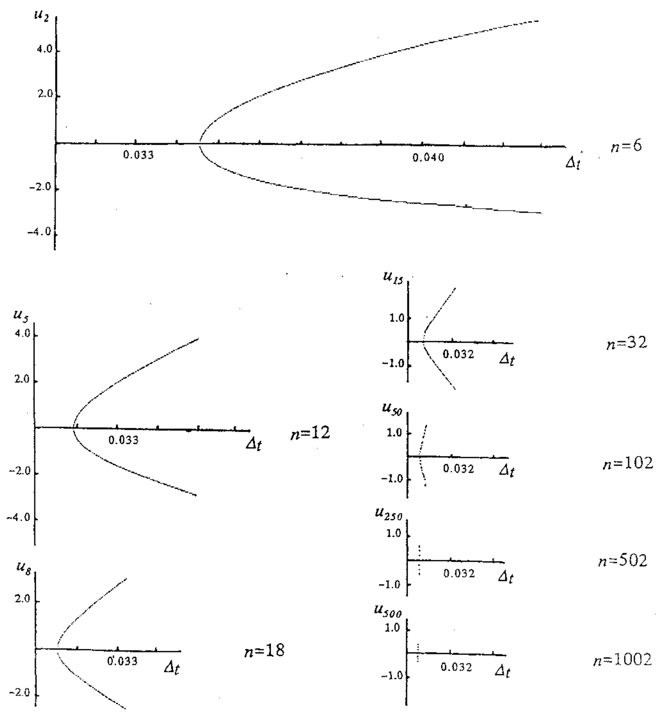

Fig. 1 Bifurcation diagrams of the $(n-2)$-points dynamical system using Eqs. (1) and (2a) discretization.

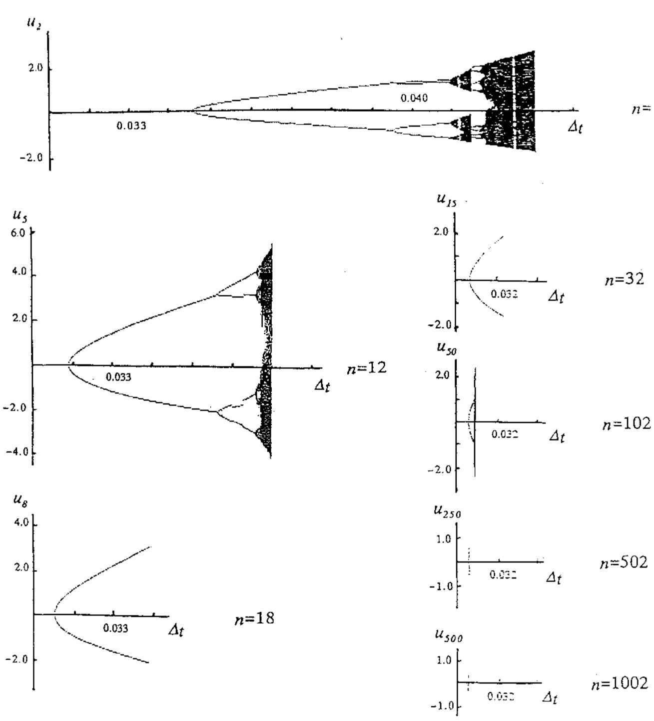

Fig. 2 Bifurcation diagrams for the $(n-2)$-points dynamical system using Eqs. (1) and (2b) discretization.

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